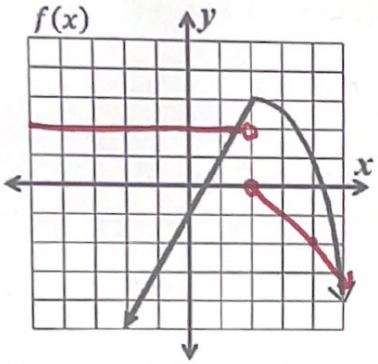
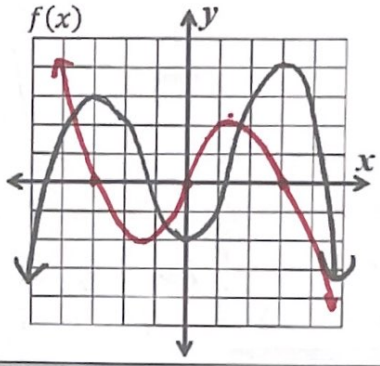
AP Calculus BC – Worksheet 35 **Analyses of graphs using f' and f''**

The graph of a function f is shown. On the same coordinate plane, sketch a graph of f', the derivative of f.

1.

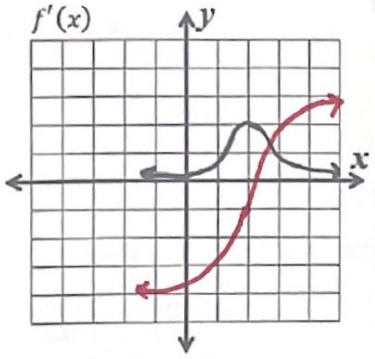


2.

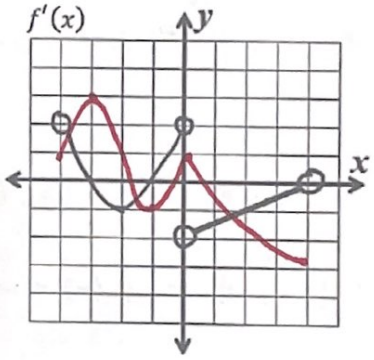


The graph of f', the derivative of f, is shown. On the same coordinate plane, sketch a possible graph of f.

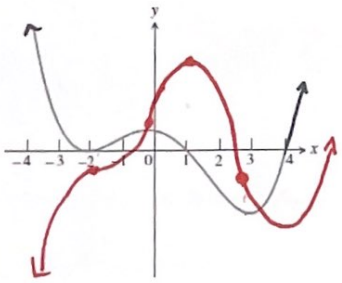
3.



4.



5. The graph of the <u>derivative</u> of f is shown below.



a) On what intervals is f increasing? Justify.

$(-\infty, -2)\ (-2, 1)\ (4, \infty) \rightarrow f'(x) > 0$

b) On what intervals is the graph of f decreasing? Justify.

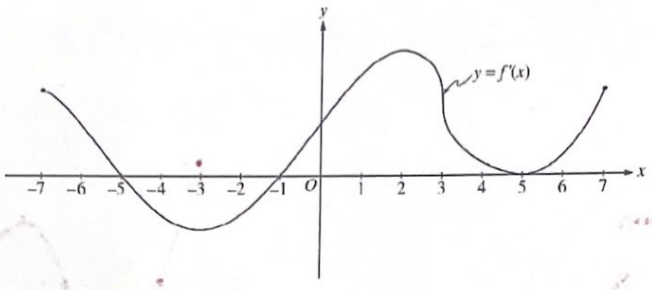
$(1, 4) \rightarrow f'(x) < 0$

c) At which x-coordinates does f have local extrema? Justify.

max @ $x = 1 \rightarrow f'$ Δs from + to −

min @ $x = 4 \rightarrow f'$ Δs from − to +

d) Sketch a possible graph of f on the interval $(-\infty, \infty)$.



6. The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. f' has zeroes at $x = -5, -1$, and 5 and horizontal tangent lines at $x = -3, 2,$ and 5.

(a) Find all values of x, for $-7 < x < 7$, at which f attains a relative (local) minimum. Justify your answer.

min @ $x = -1$ → f' Δs signs from − to +

(b) Find all values of x, for $-7 < x < 7$, at which f attains a relative (local) maximum. Justify your answer.

max @ $x = -5$ → f' Δs signs from + to −.

(c) Describe the increasing/decreasing behavior of the graph of f.

Inc : $[-7, -5)\ (-1, 5)\ (5, 7]$ → $f'(x) > 0$

Dec : $(-5, -1)$ → $f'(x) < 0$

(d) Find all values of x, for $-7 < x < 7$, at which f attains a point of inflection. Justify your answer.

f has a P.o.I @ $x = -3, 2, 5$ b/c $f''(x)$ Δs signs

(e) Describe the concavity of the graph of f.

f is concave up on $(-3, 2)\ (5, 7)$ b/c $f'' > 0$

f is concave down on $(-7, -3)\ (2, 5)$ b/c $f'' < 0$

7. Let f be the function defined by $f(x) = \frac{1}{3}x^3 - 3x^2 - 16x$. On which intervals is the graph of f both decreasing and concave down?

$f'(x) = x^2 - 6x - 16 = 0$

$(x - 8)(x + 2) = 0$

$x = 8 \quad x = -2$

f' + − + (−2, 8)

$f''(x) = 2x - 6 = 0$

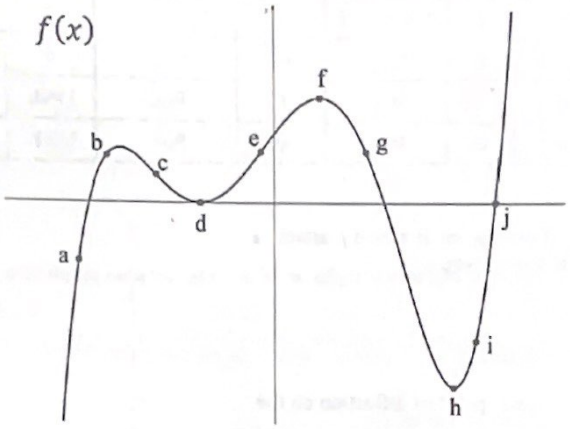
$x = 3$

f'' − + (3)

f is dec & concave down on $(-2, 3)$ b/c $f' < 0$ & $f'' < 0$

8.

Using the figure below, complete the chart by indicating whether each value is positive (+), negative (-), or zero (0) at the indicated points. For these problems, if the point appears to be a max or min, assume it is. If it appears to be a point of inflection, assume it is.



x	a	b	c	d	e	f	g	h	i	j
$f(x)$	−	+	+	0	+	+	+	−	−	0
$f'(x)$	+	+	-	0	+	0	−	0	+	+
$f''(x)$	−	−	0	+	0	−	−	+	+	+